Complete Mathematics

Problem Solving Booklet

Teacher Notes

La Salle EDUCATION

Contents

Contents

Complete Mathematics
www.CompleteMaths.com

Contents

Tangrams

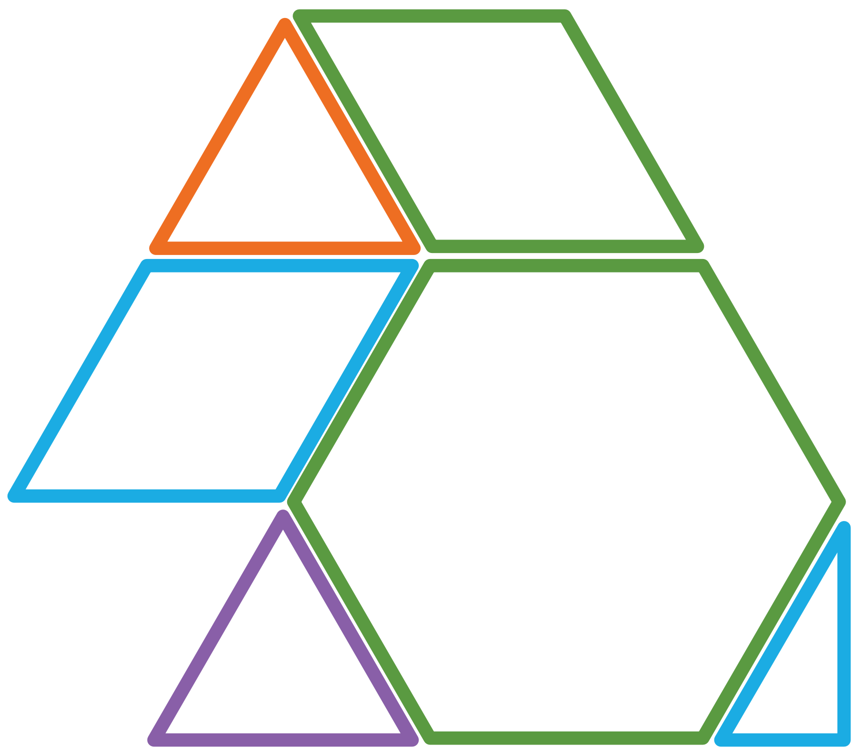
Teacher Notes

Strand: Geometry and Measures
Group: Construction, Transformation, Properties of 2D shapes, Area, Perimieter and volume
Suggested Age: 4+

This activity develops pupil ability to understand and interpret shape and space.

Pupils can sketch the shapes onto paper and perhaps cut them out and play around untill they find a way to make them fit into the outline.

Solution



10 - No More, No Less

Teacher Notes

Strand: Number
Group: Numbers and the Number System, Mental Methods of Calculation
Suggested Age: 5 - 12

You may need to point out that the boxes need to have numbers written in them, not just single digits.

The important aspect of this is the discussion around "what do you notice?" We want pupils to comment on the consistency of the ones digit.

This problem can be extended by changing the range of the solution to 20, 30... 100 etc.

To expand on this further, encourage pupils to explore different types of numbers such as fractions and decimals. Include negative numbers to produce answers like 6- -4 = 10.

One Minute Challenge 1

Teacher Notes

Strand: Number, Geometry and Measures, Probability and Statistics, Algebra
Suggested Age: 5+

This is a quick fire quiz on general Mathematical knowledge covering facts related to geometry, measure, number and data. This quiz aims to get students to remember facts and concepts when they see them out of context in a random situation. Teachers should time this activity so that it lasts for no longer than one minute, giving six seconds per fact. The ability to quickly recall mathematical facts is necessary in future time pressured situations such as exams and tests. The emboldened letters in each statement should be replaced with words. This will get students thinking, and for the final challenge to make up two further questions of their own, students should use the questions as stimuli in order to arrive at a related statement, examples are provided below.

Solution

- 12 MONTHS in a YEAR
- 60 MINUTES in an HOUR
- A PROTRACTOR is used to measure ANGLES
- V, X and C are examples of ROMAN NUMERALS
- A SQUARE has 4 RIGHT ANGLES
- This line is PERPENDICULAR
- 16, 24, 32 and 40 are MULTIPLES of EIGHT
- 1 is the NUMERATOR, 2 is the DEONOMINATOR

Examples of other statements that students may arrive at:

- A **R** has 4 **RA** – A **rectangle** has 4 **right angles**.
- A **T** has 3 **S** – A **triangle** has 3 **sides**.
- 60 **S** in a **M** – 60 **seconds** in a **minute**.

Patio Problem

Teacher Notes

Strand: Number
Group: Written Methods of Calculation
Suggested Age: 5+

Use RUCSAC
Read
Understand
Choose appropriate calculation
Solve
Answer
Check

Solutions

1. If need 78 slabs and come in packs of 12 – calculate how many packs we need.
78 divided by 12=
6 X 12 = 72
7 x 12 = 84
Therefore need to buy 7 packs as need 72

2. Cost is cost per pack multiplied by number of packs needed
7 x £25.95 = £181.65

Barry's Bricks

Teacher Notes

Strand: Number
Group: Mental Methods of Calculation
Suggested Age: 6 - 7

The problem is to make 2 moves only to rearrange 4 piles of bricks so they are all the same height.
The pupils will need to use addition, subtraction and division.

This could be done as an individual task or in small groups. Pupils should explain how they tackled the problem, and what maths they used, and what decisions and reasoning they used.

Solution

How many bricks needed in each pile?

$$\frac{6 + 3 + 5 + 2}{4} = \frac{16}{4} = 4$$

No pile is the correct height, but we can achieve the desired result in only two moves.

We must therefore be able to make two piles the correct height in one move, and then the other two piles the same height in the second.

So we can pair the piles of bricks together, and if each pair of bricks should be

$2 \times 4 = 8$

Then which piles can be paired to make a total of 8 bricks?

$6 + 2 = 8$ & $3 + 5 = 8$

So 1st move =

6 2

(6 → 2) 2 bricks

becomes...

4 4

2nd move =

5 3

(5 → 3) 1 brick

becomes

4 4

Number 7

Teacher Notes

Strand: Number
Group: Numbers and the Number System
Suggested Age: 6 - 9

This problem reinforces understanding of place value.

If pupils stick with 3 digit numbers, the answer is straight forward.

For 4 digit numbers, the statement will ‘sometimes’ be true.
Example: 4650 is greater than 1750.

Encourage pupils to write down a series of bullet points to explain when the statement is/not true.

You could ask pupils to design a poster explaining their reasoning with examples.

The problem can be made more accessible by changing the number to a 2 digit number and focusing on the tens position.

Total of 5

Teacher Notes

Strand: Number
Group: Numbers and the Number System, Mental Methods of Calculation
Suggested Age: 6+

Start by finding 2 digits which add up to 5 (remembering that there are no 0's)
1+4
2+3
3+2
4+1
Encourage pupils to use a systematic approach
Then convert these to 2 digit numbers

You could extend this activity by discussing 3 digit numbers (or maybe even more!)

What if created 3 digit number – eg 113
4 digit numbers eg 1112
Etc...

Solution

14
23
32
41

Chessboard Squares

Teacher Notes

Strand: Number
Group: Mental Methods of Calculation
Suggested Age: 7-11

Although this investigation seems quite simple, it requires a logical and systematic approach if the correct answer is to be found.

As the title suggests, the investigation involves pupils finding out how squares there are on a chessboard. They might at first think that there are only 64, (1X1 squares) so make sure to encourage them to find squares of different sizes.

It may be useful to ask how they are going to record the ones that they find and ensure that they don't record some more than once.

It would be helpful to have multi-copies of 8X8 grids for them to record the ones they find. We have provided some grids to print off if you do not have any.

Lastly when they record their working– do they see a pattern?

Solution

		The Pattern
1	8x8 square	(1^2 squares)
4	7x7 squares	(2^2 squares)
9	6x6 squares	(3^2 squares)
16	5x5 squares	(4^2 squares)
25	4x4 squares	(5^2 squares)
36	3x3 squares	(6^2 squares)
49	2x2 squares	(7^2 squares)
+ 64	1x1 squares	(8^2 squares)
204	TOTAL Squares on 8x8 chessboard	

8 different size squares

Number of squares on 10x10 chessboard =

$204 + 9^2 + 10^2 =$

$204 + 81 + 100 = 385$

Consecutive Sums

Teacher Notes

Strand: Number
Group: Number Theory
Suggested Age: 7+

This problem, once understood, can be accessed on multiple levels. At the most simplistic level, students can experiment with different sorts of numbers and should be encouraged to look for structure and order within their work. As the age/ ability of the students increases they can begin to look at addressing questions such as:

Which numbers cannot be made?

Why are the powers of 2 the only numbers which cannot be represented

What happens when you add 2 consecutive numbers?

What happens when you add 3 consecutive numbers?

Solution

All integer numbers can be expressed as the sum of consecutive integers except those numbers that are a power of 2 (1, 2, 4, 8, 16, 32, 64, etc).

Ducks, Wolves and Elephants

Teacher Notes

Strand: Probability and Statistics
Group: Data Handling
Suggested Age: 7 - 11

The activity is designed to introduce the concepts of Sample Space Diagrams which can help with finding the probability of two events happening.
Most pupils will dismiss many of the options as being duplicates from the same boxes, such as W D and W D and some will also dismiss pairings that appear the same from different boxes, such as D W and W D.

Likely incorrect answers would be 4 and 6. This can be the catalyst to discussions about why the class are split between 4,6,16 and other values.
When trying to progress from the limited list of DD, DW, EW and WW ask the pupils whether it is as likely to get two ducks as it is to get two Wolves? They will probably agree that is not reasonable as there are a lot more ducks 'flapping' around the boxes. From there they can reconsider their listings.

If you really want to convince an unbelieving audience about the 16 different combinations, offer to number the ducks and the elephants. Set out the template of a space diagram and get the pupils to complete it with you.
The investigation could lead on to extracting probabilities from your Space Diagram.

Solution

Overall there are 16 possible combinations.
They can be outlined in a Sample Space Diagram as follows:

	D	D	D	W
D	DD	DD	DD	DW
W	WD	WD	WD	WW
E	ED	ED	ED	EW
E	ED	ED	ED	EW

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How Many Sweets?

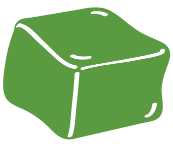
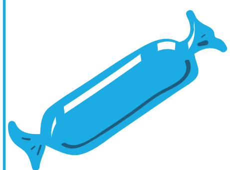
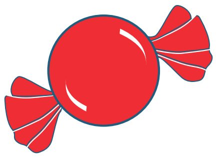
Teacher Notes

Strand: Probability and Statistics
Group: Statistics
Suggested Age: 7+

Pupils should already know that a pie chart is shared around a full turn of 360°.
Rather than working out that 10° = 4 sweets they may work out that 1 sweet = 2.5°.
This is a good point for discussion.
This question could also be extended to constructing the pie chart.

Solution

Answer is 144 sweets
Other sweets represent 360 – (90 + 135 + 60 + 45)
Other = 360 – 330 = 30°
So, if 30° = 12 sweets, then 10° = 4 sweets and 5° = 2 sweets

Sweet Type	 Jellies	 Minties	 Chockblocks	 Chewsies	Other
Pie Chart Angle	**90°**	**135°**	**60°**	**45°**	**?**
No. of Sweets	**36**	**54**	**24**	**18**	**12**

Population Pizza!

Teacher Notes

Strand: Number
Group: Number Theory & Probability
Suggested Age: 7 - 18

The question cannot actually be answered, since on closer inspection the advert does not state how many toppings each pizza should have. Since it might be possible to have as many as twenty toppings, or more, there certainly **could** be more combinations than people in the world.

A far more interesting question is ‘how many toppings would the pizza have to be able to have at least, in order to make Domino’s claim true?’

There are 26 toppings on offer.
There are approximately 7 billion people in the world.

$26^6 = 308\ 915\ 776$

$26^7 = 8\ 031\ 810\ 176$

For younger children, the problem provides a way in to start talking about combinatorics. A much simpler question for primary age children would be along the lines of “If there were only two choices of topping, pepperoni and beef if you can only have two toppings on the pizza, but double pepperoni and double beef are allowed, how many different choices of pizza are there? What if you were allowed three toppings / four / five etc. What if a new choice of mushrooms was added? etc.”

For younger secondary aged children, it might be worth discussing the fact that the question can’t be answered together, first, and discuss assumptions like ‘having as many of one topping as you like would have to be allowed,’ then setting them off to calculate how many toppings would be required on their own.

For older secondary aged children, it might be possible to leave them to realise the impossibility for themselves, and they should increasingly be expected to state assumptions

Continued...

Population Pizza!

For A Level students, this can provide an opportunity to practice combinatorics if they were taught it at school, or else can serve as a problem to study during combinations and permutations, and an opportunity to show the utility of logarithms; while this problem can be easily solved using trial and improvement, the equation that models the problem is:

$26^x = 7\ 000\ 000\ 000$

This can be solved using logs:

$$x = \frac{\log 7 \text{ bn}}{\log 26}$$

$x = 6.96$

So 7 toppings are needed.

Samantha's Sweets

Teacher Notes

Strand: Algebra
Group: Equations
Suggested Age: 7 - 14

At the lower end of the age range the students will work towards a solution using discussion or maybe concrete objects.

Towards the higher end of the age range students will be expected to form and solve an algebraic equation.

Solution

This can be solved using trial and improvement or using an algebraic method.

Let n represent the number of sweets Samantha has to begin with.

Steven:

$$\frac{1}{2}n - 2 = 4$$

$$\frac{1}{2}n = 6$$

$$n = 12$$

The 100 Quiz

Teacher Notes

Strand: Number, Geometry and Measures
Group: Written Methods of Calculation, Number theory, Measures, Time
Suggested Age: 7+

The questions in this quiz relate to the number 100, and are split into sections, from easy through to more challenging questions requiring working out. The quiz tests knowledge of mathematical facts, the ability to work out square roots, factors, prime factors, and to understand binary form.

Primary phases or lower ability classes can attempt the easy and medium level questions whilst KS 3 & 4 classes will be able to try the harder questions. There are points available for answers, with a maximum score of 22 points.

Solution

Easy – One point each

100 pence = **£1**
100cm = **1 metre**
100 years = **1 century**
The total amount = **100%**

Medium – Two points each

What letter represents 100 in Roman Numerals? **C**
$\sqrt{100}=$ **10**
True or false; the sum of the first nine prime numbers is 100? **True**
2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 = **100**

Hard – Three points each

Find all the factors of 100 - **1,2,4,5,10,20,25,50,100**

Show the prime factors of 100 – **2 x 2 x 5 x 5**

Show the prime factors of 100 in exponential form – $2^2 \times 5^2$

Show the number 100 in binary form - **1100100**

Totalling Odd Numbers

Teacher Notes

Strand: Number
Group: Written methods of calculation
Suggested Age: 7-8

Ensure all are familiar with odd numbers. Remind them, if necessary, and refer to displays if appropriate.

1 + 3 + 5 + 7 + 9

Answer is 25

Was there an easy way to work out answer? Did they see the number bonds to 10?

What if add first 20 odd numbers.

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 25 (from 1st 5 numbers) + 50 (tens digits on next 5 numbers) + 25 (units digits of 2nd 5 numbers) = 25 + 75 (25+50)
Look at first 30 odd numbers
Can they predict the answer?
(0x5) + 25 + (10x5) +25) + ((20x5) + 25) = 225

Solution

Total of all odd numbers less than 10 is 25
Total of all odd numbers less than 20 is 125
Total of all odd numbers less than 30 is 225

Wheelzr'us

Teacher Notes

Strand: Number
Group: Written Methods of Calculation
Suggested Age: 7 - 12

Pupils may notice that the number of Trikes must go up in 2's as 3 wheels is an odd number and cannot be replaced by a bike which has an even number of wheels.

There are many solutions

Younger pupils may only find one solution, but older pupils should be encouraged to find other solutions and look for patterns in terms of the trikes being in 2's.

Young children could be prompted to start with 30 bikes or 20 trikes

Solution

Number of Bikes	Number of Trikes	Bike Wheels	Trike Wheels	Total
30	0	60	0	60
27	2	54	6	60
24	4	48	12	60

etc...

7's in 999

Teacher Notes

Strand: Number
Group: Numbers and the Number System
Suggested Age: 8+

This problem generates work on place value and working systematically, as well as group work and justification.

Solution

Number of 7's in the...

Units column for every increase in 10 = 1
Number of 10s in 999 = 100 (rounded)
Therefore 100 7's in the units column

Tens column for every increase in 100 = 70, 71, 72... 79 = 10
How many 100s in 999 = 10 (rounded)
Therefore 100 7's at place value ten

Hundreds column for every increase in 1000 = 700, 701... 799 = 100
Therefore 100 7's at place value hundred

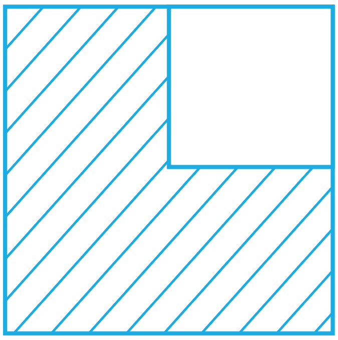
Total
= 100 + 100 + 100
= 300

Amy's Cake

Teacher Notes

Strand: Number, Geometry And Measures
Group: Fractions and Decimals, Properties of 2D Shapes
Suggested Age: 8+

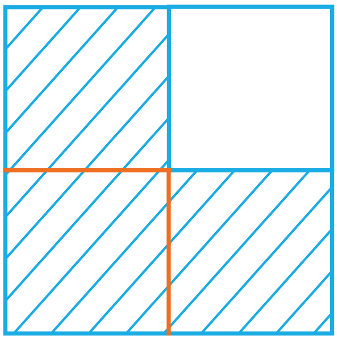
Thinking about the shape we started with, it may help to start by drawing the shape from the top down.



We know the cake is square, and that one-quarter has been removed.

We can infer from the image that it is most likely a smaller square that was removed from the cake.

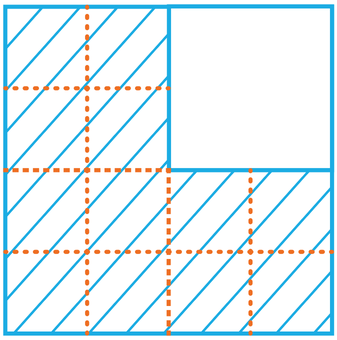
If we only had to divide the remaining shape into 3, it would be easy a little easier, as we can see the remaining shape is 3 perfect squares.



Some more visual pupils may simply understand how to break up the shape in their head, as shown in the solution.

Others may need to be guided with a more structural approach, as per the solution.

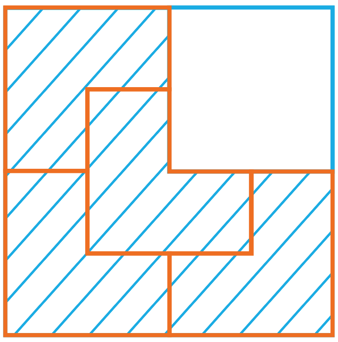
Solution



We need to make 4 equally sized pieces from the remaining 3 quarters. So we could start by saying;

"If we could slice up the cake as small as we wanted, what would be a quick way of dividing the remaining 3 squares, into 4 equal shares?'

As shown, if each square was quartered again, we could give 3 quarters of each of the smaller squares to each of the four people.



As we start to break the cake down into its constituent shapes like this, we can begin to see there is a way of cutting up the cake into four equal parts.

Car Percentage

Teacher Notes

Strand: **Number**
Group: **Fractions and Decimals**
Suggested Age: **8 - 12**

Pupils may use a variety of methods to solve this problem. This can be a useful discussion of the “best” method.

The question could be made more difficult using different values.

You could ask students to find intermediate values.

You could add on VAT, road tax, etc. and ask students for those values.

Solution

a. $3200 = \frac{2}{3} \times 0.75 \times x$, where x represents the original cost.

$3200 = 0.32\,x$

$10000 = x$

Karen paid £10,000.00

b. $\frac{3200}{10000} = $ **32%**

Cutting Corners

Teacher Notes

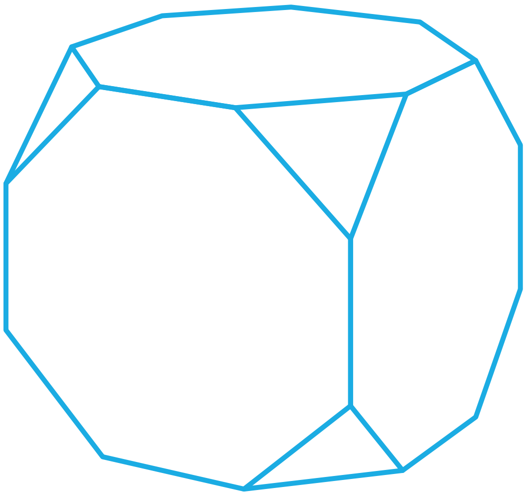
Strand: Geometry and Measures
Group: Properties of 3D shapes
Suggested Age: 8+

This is done with visualisation, without the need for models or drawing.

Asking students to justify & explain how they saw it can be fruitful.

Solution

The new shape is a truncated cube.



A truncated cube has 24 vertices.

More information about truncated cubes can be found on Wikipedia at

http://en.wikipedia.org/wiki/Truncated_cube.

Farmer Bob Goes To Market

Teacher Notes

Strand: Number
Group: Proportional Reasoning
Suggested Age: 8+

In this activity students will use problem solving skills to get to the solution. They will have to be able to think a move or two ahead and use logical reasoning to get the number of sacks of grain. They should be aware that it is the minimum number of grain sacks they are looking for. Students should remember they need one of each animal to take home.

Solution

Answer = 16
For 1 cow Farmer Bob needs to exchange goats and grain to get 5 piglets.
This means he needs 2 goats and 4 sacks of grain = 6 piglets (5 for a cow and 1 to take home).
Since he only takes grain to the market he needs 8 sacks for 2 goats.
This is a total of 12 sacks of grain for 1 cow and the 1 piglet left over.
To bring a goat home Farmer Bob will need another 4 sacks of grain.
Hence, Farmer Bob will need to bring at least 16 sacks of grain to the market.

Four 4s

Teacher Notes

Strand: Number
Group: Numbers and the Number System & Mental Methods of Mental Calculation
Suggested Age: 8 and up

This is an activity that practices order of operation and calculation, along with logical thinking and reasoning

Solution

wikipedia.org/wiki/four_fours#solutions

How Low Can You Go?

Teacher Notes

Strand: Number
Group: Numbers and the Number System
Suggested Age: 8 and up

This problem is ideal for pupils to practice their calculations and order of operations. There is also plenty of opportunity for discussion and justification of their solution.

Solution

4

New Kettle

Teacher Notes

Strand: Number
Group: Percentages
Suggested Age: 8 - 11

Pupils should be able recognise the per cent symbol (%) and understand that per cent relates to “number of parts per hundred”, and write percentages as a fraction with denominator hundred, and as a decimal fraction. They should solve problems involving the calculation of percentages (for example, of measures, and such as 15% of 360) and the use of percentages for comparison.

Repeated percentage change where the magnitude of the percentage differs for each of the subsequent changes is a common problem in ‘real life’. In both financial matters and natural phenomena so pupils - especially those seeking to progress to higher levels of maths - need to have experience in this topic.

To incorporate some unit change of units you could use 2 litres in the question.

Solution

Answer is 1800 ml so **Bob is wrong**.

20% of the 80% is 16% of the kettle’s capacity. Therefore, the volume of water left in the kettle after Bob has poured out 20% of the original amount is 64% of the total kettle capacity.

Therefore, when full the kettle holds (1152/64) × 100 = 1800 ml

The Toy Shop

Teacher Notes

Strand: Number
Group: Written methods of calculation
Suggested Age: 8 - 9

The object is to work out how many tricycles and go-carts are in the toy shop, starting from the total number of wheels.
Tricycles have 3 wheel and go-carts have 5 wheels. There are 51 wheels altogether, and pupils have to find 3 ways of solving the problem.

Pupils should explain how they tackled the problem, what reasoning they used and what maths they used.

This needs a systematic approach to work out all the combinations of exactly 5+3 that can be made from 51.

Solution

9 go-carts and 2 tricycles
6 go-carts and 7 tricycles
3 go-carts and 12 tricycles.

Teacher Notes

Strand: Number
Group: Mental Methods of Calculation, Written Methods of Calculation, Fractions and Decimals, Percentages
Suggested Age: 9 - 15

This apparently simple question can work across a wide ability range of pupils. The task can span a full lesson and homework by asking pupils to explore extension tasks such as introducing new currencies or not supplying them with the values of each item and asking them to undertake research to determine suitable values.

For many, the leap of recognising that there are not 12 gifts in total will take some time. It will help to play the song to the pupils so that they appreciate the repetition of gifts each verse.

The values given are deliberately units – so rather than a cost of 'eight maids a-milking', the pupils are told the price of just one maid! Again, working out the total figures for the gift, in this case eight maids, will be a trivial activity for some but a challenging activity for others. You can decide whether or not to allow calculators for this stage, which again can introduce new challenge.

Once the costs of a single instance of each 'gift' is calculated and the number of occasions this gift is given, it is a simple case of multiplying and summing the results to get the total cost of the 12 Days of Christmas – but look out, the values are all in US Dollars. There is now a conversion activity to carry out. In our solutions, we have used 1 USD = 0.64 GBP. You could extend the task by converting to other currencies or by changing the conversion factor, which might lead to discussions around why currencies do not remain constant relative to each other.

The first table shows the calculations to find the total cost in GBP. The pupil pages include a blank table that pupils can use to order their work. Alternatively, you might wish to extend the task by not providing this table and asking the pupils to come up with a sensible and efficient way of presenting their work.

Question 3, asks pupils to work out the total cost in 2013. They are provided with the percentage increase this year from last and will need to use this information to calculate the values for 2013. Mostly, the individual items have 0% increase. You might wish to ask the pupils why this might be – this is not often the case in the history of the Christmas Price Index, but we are currently experiencing very low inflation globally.

Many pupils will initially make errors in converting the prices, by assuming that a percentage increase is undone by a percentage decrease of the same amount. Of course, this is not the case. You could point this out to them beforehand, or let them make the errors first and try to work out where they went wrong. For example, pupils may try to calculate the 2013 cost of a Partridge in a Pear Tree by reducing $207.68 by 3.8% rather than recognising that the initial increase was achieved by multiplying by 1.038 and therefore, reversing this change requires a division by 1.038.

Solution

Data from the 2014 PNC Christmas Price Index
https://www.pncchristmaspriceindex.com/
Calculating total cost in 2014

	Gift	Unit Cost 2014	Cost per gift 2014	Variance on 2013	Unit Cost 2014 UK	Cost per gift 2014 UK	Days	Cost	Total number of Gifts
1	Partridge in a pear tree	$207.68	$207.68	3.80%	£132.92	£132.92	12	£1,594.98	12
2	Two turtle doves	$62.50	$125.00	0.00%	£40.00	£80.00	11	£880.00	22
3	Three French hens	$60.50	$181.50	10	£38.72	£116.16	10	£1,161.60	30
4	Four calling birds	$149.99	$599.96	0%	£95.99	£383.97	9	£3,455.77	36
5	Five gold rings	$150.00	$750.00	0%	£96.00	£480.00	8	£3,840.00	40
6	Six geese a-laying	$60.00	$360.00	71%	£38.40	£230.40	7	£1,612.80	42
7	Seven swans a-swimming	$1,000.00	$7,000.00	0.00%	£640.00	£4,480.00	6	£26,880.00	42
8	Eight maids a-milking	$7.25	$58.00	0%	£4.64	£37.12	5	£185.60	40
9	Nine ladies dancing	$839.20	$7,552.84	0%	£537.09	£4,833.82	4	£19,335.27	36
10	Ten lords a-leaping	$534.82	$5,348.24	2%	£342.29	£3,422.87	3	£10,268.62	30
11	Eleven pipers piping	$239.56	$2,635.20	0.00%	£153.32	£1,686.53	2	£3,373.06	22
12	Twelve drummers drumming	$237.90	$2,854.80	0.00%	£152.26	£1,827.07	1	£1,827.07	12
			1.00 US dollars =	0.64	British Pounds			£74,414.77	364

12 Days of Christmas

	Gift	2014 Cost	Divisor	2013 Cost
1	Partridge in a pear tree	£1,594.98	1.0380	£1,536.59
2	Two turtle doves	£880.00	1.0000	£880.00
3	Three French hens	£1,161.60	1.1000	£1,056.00
4	Four calling birds	£3,455.77	1.0000	£3,455.77
5	Five gold rings	£3,840.00	1.0000	£3,840.00
6	Six geese a-laying	£1,612.80	1.7140	£940.96
7	Seven swans a-swimming	£26,880.00	1.0000	£26,880.00
8	Eight maids a-milking	£185.60	1.0000	£185.60
9	Nine ladies dancing	£19,335.27	1.0000	£19,335.27
10	Ten lords a-leaping	£10,268.62	1.0200	£10,067.28
11	Eleven pipers piping	£3,373.06	1.0000	£3,373.06
12	Twelve drummers drumming	£1,827.07	1.0000	£1,827.07
			Total 2013 Cost	£73,377.59

1. How many gifts in total would you receive in the entire song?
   364

2. The individual prices are shown in US dollars, what is the full cost in UK pounds?
   £74,414.77

3. What would the total cost in 2013 have been in UK pounds?
   £73,377.59

4. What is the total percentage increase from 2013 to 2014?
   1.41%

150 Dice

Teacher Notes

Strand: Number
Group: Written methods of calculation
Suggested Age: 9+

The top and bottom number of a dice always add up to 7.
150 dice will have a total of 1050 (150 x 7) if you add up top and bottom together.
If current score is 441 then the remaining score for the bottom is 1050-441=609

Solution

609

Aircraft Luggage

Teacher Notes

Strand: Number
Group: Mental Methods of Calculation & Written Methods of Calculation
Suggested Age: 9 - 11

This problem requires pupils to recognise they need to find the allowable weight.

They need to consider how 40kg produced a change of £50 and produce the calculation to show what the allowable weight is.

This can then be used to work out the second situation.

Solution

A £50 charge represents 5kg overweight as 5 × £10 = £50 charge,
40kg - 5kg = 35kg for allowable weight.

Therefore 80kg - 35kg = 45kg overweight and the passenger would be charged
45 × £10 = £450

Decimal Disposition

Teacher Notes

Strand: Number
Group: Numbers and The Number System
Suggested Age: 9 - 12

Note – a single digit in each box

Encourage pupils to do some thinking and talking before they start this problem.
Some key prompt questions:

Which are the larger digits?
Which is more important in terms of size, the 1s digit or the 1/10 digit?

There are twelve possible solutions.
Encourage a systematic approach.

You can adapt the content of this problem by:

Removing the decimal point and using 2 digit integers.
Adding a second place of decimals.

Solution

□ . □ > □ . □

3	.	7	2	.	8
3	.	8	2	.	7
7	.	2	3	.	8
7	.	3	2	.	8
7	.	8	2	.	3
7	.	8	3	.	2
8	.	2	7	.	3
8	.	2	3	.	7
8	.	3	7	.	2
8	.	3	2	.	7
8	.	7	3	.	2
8	.	7	2	.	3

Just Make One

Teacher Notes

Strand: Number
Group: Written methods of calculation
Suggested Age: 9+

Remind pupils about the use of brackets and BIDMAS:
Brackets
Indices
Division
Multiplication
Addition
Subtraction

Solution

$(6 \times 6) - (7 \times 5)$

Mathematical Mind Reader

Teacher Notes

Strand: Number
Group: Number Theory
Suggested Age: 9+

Remind your pupils what factors are.
How do you find factors of a number?
If the number only has 2 factors – what do we call it?

Think about factors of 36.
Write down these factors.
Now Re-read the question - what do we have to find out?

Solution

The factors of 36 are

1, 2, 3, 4, 6, 9,12, 18, 36

The odd numbers are 3, 9

Only 9 is larger than 5

Meet The Joneses

Teacher Notes

Strand: Number
Group: Numbers and the Number System
Suggested Age: - 9 +

This problem requires knowledge of prime numbers, the four operations and an ability to sensibly work out the steps needed to go from a word problem to the calculations needed in order to solve it. Pupils will have to consider where is the best starting point, which piece of information is the most useful? Pupils will need to carefully read the language used in the problem, and check back to the problem during the working out.

Solution

Starting with the prime numbers below 20, there are several combinations that have a difference of 6, to give the ages of Tom and Georgia.

5 and 11

7 and 13

11 and 17

13 and 19

If Tom, the youngest, is a quarter of the age of his father, multiply to find possible ages of Mr Jones:

5 x 4 = 20 Less than 40, this possibility is excluded.

7 x 4 = 28 Less than 40, this possibility is excluded.

11 x 4 = 44

13 x 4 = 52

Mrs Jones is four years younger than Mr Jones, subtract to find possible ages of Mrs Jones:

44 – 4 = 40 Exactly 40, not greater than 40, this can now be excluded.

52 – 4 = 48

Now you can confirm the ages of four members of the family:

Mr Jones is 52

Mrs Jones is 48

Georgia is 19

Tom is 13

Finally, find the age of the eldest, Fiona, she is half the age of her mother.

48 ÷ 2 = 24

Fiona is 24.

Multiplying This Way and That

Teacher Notes

Strand: Number
Group: Written methods of calculation
Suggested Age: 9 - 12

Encourage the use of visual images to help; maybe make a link with area. For example, a floor that measures 73 m by 13 m will always have the same area.
Encourage estimation first to ensure answer is reasonable.
Consider different calculation methods such as grid or long multiplication.

Solution

Multiplication is commutative, i.e. it doesn't matter in which order you multiply the values, the answer will be the same.

$73 \times 13 = 578$

Passcode Combinations

Teacher Notes

Strand: Number
Group: Numbers and the number system
Suggested Age: 9+

First number of the 4 could vary between 9 numbers- 1, 2, 3, 4, 5, 6, 7, 8 or 9
If the passcode begins with 1, the second number could be 1 out of 10 numbers- 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
So variation for 2 numbers beginning with 1 is 10
If third number has a variation of 10 then each number above e.g. 10, 11, 12, 13 etc has a variation of 10 e.g. 101, 102, 103 etc
So for 3 numbers:-
Total number = 1 x 10 x 10
Same for the 4th number
So if the number begins with 1, the variations are 1 x 10 x 10 x 10 = 1000
But there are 9 different ways of starting the passcode so total = 9 x 1000 = 9000

Solution

9000

Triathlete Pete

Teacher Notes

Strand: Geometry and Measures
Group: Measures
Suggested Age: 9+

We need all lengths to be in the same units so we can add up the total length of the race so far.

Check an understanding of the fact that 1km = 1000m

Encourage pupils to record the individual totals.

Solution

1. 2 × 750 metres = 1500 metres
2. 5 km = 5000 metres
3. 1250 metres
4. ??

1500 + 5000 + 1250 = 7750
10 km = 10000 metres 10000 - 7750 = 2250 metres
The finishing distance on the track = 2250 metres
Each lap is 750 metres so we divide 2250 by 750
2250 ÷ 750 = 3

So we need 3 laps of the track to finish the race.

27 - No More, No less

Teacher Notes

Strand: Algebra
Group: Equations
Suggested Age: 10 - 13

What numbers add up to make 27?
Can they use this as a starting point?
Is there a logical approach to finding the other answers?
Is the number of pairs found linked to the answer?
Do all odd totals have the same number of pairs?
Do all the even totals have the same number of pairs?
Is there a link between the total and the pairs in terms of odd and/or even numbers?

Solution

7 + 20	(odd + even)
8 + 19	(even + odd)
9 + 18	(odd + even)
10 + 17	
11 + 16	
12 + 15	
13 + 14	

Bunch of Roses

Teacher Notes

Strand: Number
Group: Fractions and Decimals
Suggested Age: 10 - 12

This exercise will develop pupil ability to approach and break down multi-step problems to find a solution.

Solution

12 × 1.12 = 13.44

30 - 1.44 = 28.56

28.56 - 13.44 = 15.12

15.12 ÷ 1.89 = 8

Laura bought 8 roses.

Carlos' Coloured Crate

Teacher Notes

Strand: Geometry And Measures
Group: Properties of 3D shapes
Suggested Age: 10+

This problem is abstract, but a knowledge of nets and elevations will be an advantage. Students should also know the cube has 6 identical faces and they should know the terms faces, edges and corners (vertices).
It may help them to see the problem better if they draw out the net of a cube.

Solution

Answer is 3
Consider taking one corner, three of the faces meet there. Each pair has an edge in common so we need three different colours. There are 6 faces on the cube and provided that Carlos paints the opposite faces in the same colour he will need no more colours, as opposite faces do not share an edge.

Fascinating Factors

Teacher Notes

Strand: Number
Group: Written Methods of Calculation
Suggested Age: 10+

Ensure an understanding of the term **factors**, there is often confusion between factors and **multiples**

Definition of a Factor:
When a number, or polynomial in algebra, can be expressed as the product of two numbers or polynomials, these are factors of the first.

Definition of a Multiple
For any integers a and b, a is a multiple of b if a third integer c exists so that $a = bc$

Encourage the listing of factors using a logical approach, this will ensure that all factors are included.

It may be useful for some students to have the use of a multiplication grid to help them find the factors.

Solution

(i) 36

(ii) Using 'Factor Pairs', we have

1 × 36
2 × 18
3 × 12
4 × 9 and
6 × 6

Be careful not to include '6' twice

so we have 9 factors - 1, 2, 3, 4, 6, 9, 12, 18, 36

Football or Rugby?

Teacher Notes

Strand: Number
Group: Percentages, Fractions and Decimals
Suggested Age: 10+

a) In order to change from fraction to percentage, we need to understand what is meant by %.

% means out of a hundred. Therefore, we need to convert $\frac{3}{5}$ to $\frac{?}{100}$

Whatever we multiply the denominator of $\frac{3}{5}$ by to get the answer 100, is what we need to multiply the numerator by.

We need to multiply 5 by 20 to get 100, therefore we need to multiply 3 by 20 as well.

Therefore is $\frac{3}{5}$ equal to $\frac{60}{100}$ = 60%

b) We then need to find the number of people who prefer Football to Rugby. So we need to calculate 60% 0f 200. To find 10% we divide 200 by 10 = 20, If 10% is 20, we multiply 20 by 6 to find 60%

Solution

a) 60%
b) 40×3=120
60% or 120 people prefer Football to Rugby

Garden Greenhouse

Teacher Notes

Strand: Geometry and Measures
Group: Area, Perimeter and Volume
Suggested Age: 10+

Solution

1. First you will need to work out area of the grey area.
   To do this you need to calculate length A and B.
   It can be seen that A + 9m = 15m, therefore A = 15m - 9m.
   So A = 4m
   In the same way B + 22m = 33m, therefore B = 33m – 22m.
   So B = 11m

   Therefore area is 4 × 11 = **$44m^2$**

2. To calculate the remaining space you need to work out the total area of the garden minus the area of grey area.

   Area of total garden is 15m × 33m = 495 m^2
   We know that the area of the greenhouse is $44m^2$
   So Area of the remaining space is
   495 m^2 – $44m^2$ = **$451m^2$**

Granny's Birthday

Teacher Notes

Strand: Number
Group: Number Theory
Suggested Age: 10+

Pupils should already know square numbers from 1 -100 and what defines a prime number.
Note that since all prime numbers other than 2 are odd, the only square numbers which need to be checked, other than 1 are of even numbers.

Solution

Answer is 4 times
That is from 1 to 2; from 4 to 5; from 16 to 17 and from 36 to 37.

Ice-cream Combinations

Teacher Notes

Strand: Algebra
Group: Equations
Suggested Age: 10+

Pupils can underestimate the number of different combinations that can be made from a set of items. They can also get confused as to when switching the order of a pair of items produces a different pair to when it doesn't. There will be opportunities for pupils to explore different techniques to select pairs from a list and develop a systematic approach e.g. keep the first item the same and changes the second.

Encourage pupils to record combinations visually if needed.

A logical approach is recommended - working through 1 type of ice cream with each type of topping. Rather than recording by longhand encourage shorthand.

E.g.

Strawberry	-	s
Chocolate	-	ch
Coffee	-	c
Vanilla	-	v
etc.		

Combinations are
S & sp
S & f
S & sa

Repeated for 3 other flavours.

Solution

12 combinations

Order! Order!

Teacher Notes

Strand: Number
Group: Written methods of calculation
Suggested Age: 10 - 16

The first challenge for students is to be able to equate this sum correctly without the brackets. Even older pupils often neglect the order of operation that needs to apply. Asking pupils to find a value for this sum will lead to numerous different answers and a lot of discussion opportunities.

Having then established the order of operations, pupils can try prioritising operations to firstly maximise and minimise the value.

Younger students could be allowed calculators to explore this, which would give them the opportunity to use the bracket function on scientific calculators.

Pupils should quickly realise that putting brackets around multiplication or division sums does nothing to the order of operation.

Solution

The value of this sum is 45

Other values are obtained as follows:

$(4+3)^2\times5-8\div2=241$

$(4+3^2)\times5-8\div2=61$

$4+(3^2\times5-8)\div2=22.5$

$(4+3^2\times5-8)\div2=20.5$

$4+3^2\times(5-8\div2)=13$

$4+3^2\times(5-8)\div2=-9.5$
(a tricky one to calculate)

Allowing two brackets certainly can bring the value lower, for example:

$(4+3)^2\times(5-8)\div2=-73.5$

It is not possible to make a higher number using 2 sets of brackets.

Complete Mathematics
www.CompleteMaths.com

Petrol Tank

Teacher Notes

Strand: Number
Group: Mental Methods of Calculation
Suggested Age: 10+

a. How to find a quarter of an amount. It is possible to either halve the amount (divide by 2) and then halve the answer (divide by 2) or divide the amount of petrol by 4

b. To find the amount left – subtract the answer to part a) from the initial amount of petrol in the car, or find $\frac{3}{4}$ of 38. You can do this by finding $\frac{1}{4}$ and then multiplying the answer by 3.

Solution

a. 38 divided by 4 = $9\frac{1}{2}$ litres

b. $38-9\frac{1}{2} = 28\frac{1}{2}$ litres

Ralph’s Sweets

Teacher Notes

Strand: Number
Group: Number and Place Value
Suggested Age: 10+

Consider writing the names of the sweets on bits of paper – this will = allow them to be moved around.

Check understanding of language in this question– for example ‘between’ is not the same as next to!

Which is the fixed piece of information? The Blue Sweet comes second.

Solution

Orange Sweet
Blue Sweet
Green Sweet
Red Sweet
Purple Sweet

Reversing Numbers

Teacher Notes

Strand: Number
Group: Numbers and the number system
Suggested Age: 10+

This is quite a famous pattern in mathematics, and it is a great way to show pupils that there is far more to maths than meets the eye!

This is a great activity to do with the whole class, all at once. Tell them not to peek at each others, and that at the end you will count them down; at which point they can all say their number out loud. You might even wish to write the number 1089 and then reveal it at the end to show you always new what number they would end up with.

The next step is to ask the pupils if they will always get this number with any three digit number. Higher ability students may be able to work out the solution if given twenty minutes or so.

Solution

Let's take our three numbers as: a, b, c

we know that if we are ordering them biggest to smallest, then: $a > b > c$

we know that $a > c$ so we are going to need to borrow ten from the tens column in step 2

1

	a	b	c
-	c	b	a

2 (borrow 10)

		b-1	c+10
	a	~~b~~	~~c~~
-	c	b	a
			c + 10 - a

3 (borrow 10)

		b+9	
	a -1	~~b-1~~	c+10
	~~a~~	~~b~~	~~c~~
-	c	b	a
	a - 1 - c	9	c + 10 - a

4

		a - 1 - c	9	c + 10 - a
+		c + 10 - a	9	a - 1 - c
	1	0	8	9

5 (carry 10)

		~~a~~ - 1 - ~~c~~	9	~~c~~ + 10 - ~~a~~
+		~~c~~ + 10 - ~~a~~	9	~~a~~ - 1 - ~~c~~
	1	0	8	9
		1		

Two Jugs

Teacher Notes

Strand: Number, Geometry and Measure
Group: Proportional Reasoning, Measures
Suggested Age: 10+

In this activity students will have to use problem solving skills to get to the solution. They will have to be able to think a move or two ahead and use logical reasoning to get the required amount of water.

They should know that the jugs will only hold the amount stated and that they may not fill the jugs other than to fill them completely.
To extend each one of the problems they should try to complete the solution in the smallest amount of moves.

They could also work in pairs designing their own and letting their partner solve it.

Solution 7L + 5L Jugs

Fill 7L jug and pour into the 5L jug.
There is now 2L in the 7L jug.
Empty the 5L jug.
Pour the 2L of water into the 5L jug.
Fill the 7L jug and pour into the 5L jug. (This will only take 3L as there is already 2L in)
Therefore, there will be 4L left in the 7L jug.
There could be other solutions discuss these with the students. Maybe compare the number of moves.

Solution 11L + 6L Jugs

Fill the 6L jug and pour into 11L jug.
Fill 6L jug again and tip into 11L jug – this will leave 1L in the 6L jug.
Empty the 11L jug
Pour the 1L into the 11L jug.
Fill the 6L jug again and pour into the 11L jug to make 7L now.
Fill the 6L jug again and pour into the 11L jug. This will only take 4L as there is already 7L in. There will be 2L left in the 6L jug.
Empty the 11L jug again.
Pour the 2L into the 11L jug and pour on another 6L = 8L

A Date with Cubes

Teacher Notes

Strand: Number, Geometry and Measures
Group: Number Theory, Numbers and the Number System, Properties of 3D shapes.
Suggested Age: 11 - 16

This problem requires knowledge there is 31 days maximum in a month so combinations from 01 to 31 are required. There are various approaches to solving this ranging from trial and error to working out the combinations needed and logically inferring which numbers are needed.

Part of the problem is to realise a 6 can be turned upside down to be a 9. Higher ability pupils may logically deduce the answer from what must be needed to make all the combinations work.

Solution

The highest numbers needed are 30 and 31. This means only one cube with a 3 on is required.
To create 11 and 22 both cubes need a 1 and 2 on them.

A 0 is required for 01, 02, 03, 04, 05, 06, 07, 08 so both cubes have to have a 0 on them.

Each cube must have 0, 1 and 2 painted on it Leading to the solution:

Cube 1	Cube 2
0	0
1	1
2	2
3	6
4	7
5	8

The trick being, the 6 can be turned around to make 9.

Chocolate Crate

Teacher Notes

Strand: Number, Geometry and Measures
Group: Written Methods of Calculation, Time, Area Perimeter and Volume
Suggested Age: 11 to 15

There are different ways this can be worked out. After changing the crate measurements to cm, the number of bars can be found from working out how many fit lengthwise, widthwise and heightwise, then multiplying to produce amount.

Alternatively the volume of the crate and the volume of the bar can be calculated then divided.

This will require students to know 100cm = 1m and how to find the volume of a cuboid.

For the timescale they can assume 365 days in a year but higher ability may realize 365.25 days in a year is better to account for leap years.

Either way it is 82 years to the nearest year

Solution

Method 1:

Converting to cm,

Crate = 200cm by 200cm by 300cm.

Choc = 10cm by 2cm by 1cm

That is 20 bars length by 100 bars wide by 300 bars high

20 x 100 x 300 = 600’000 bars

Method 2:

Crate 200 x 200 x 300 = 12’000’000cm^3

Choc 10 x 2 x 1 = 20 cm^3

$\frac{12000000}{20}$ = 600’000 bars

$\frac{600000}{20}$ = people is 30000 bars each

$\frac{30000}{365}$ = 82 rounded to nearest year

Chocolate, Sweet or Toffee?

Teacher Notes

Strand: Probability and Statistics
Group: Probability And Statistics
Suggested Age: 11+

Work out probability of picking out each type of sweet

Start by working out how many sweets in total. 12+40+48=100

The probability of selecting a boiled sweet is $\frac{12}{100}$

The probability of selecting a chocolate is $\frac{40}{100}$

The probability of selecting a toffee is $\frac{48}{100}$

It is more likely that a chocolate or toffee will be picked out because there are more of them in the box. However, there is still a chance that Martin could win because 12 out of 100 sweets are boiled sweets.

Solution

Martin is wrong. It is not true to say that he will never win.
It would be more accurate to say that Martin is unlikely or less likely to pick out a boiled sweet

Circular Primes

Teacher Notes

Strand: Number
Group: Number Theory
Suggested Age: 11+

This problem lends itself to consideration of what makes a number prime. Realising that the number can't contain any even numbers or 5's limits options significantly

Solution

113 131 311

197 179 791 719 917 971

199 991 919

337 373 733

Consecutive 5

Teacher Notes

Strand: Algebra
Group: Sequences
Suggested Age: 11 up

It is likely that students of most ages and ability will begin by trialing a few examples. It is also likely that once they have found a few examples that a work, a claim of "I've proved it!" will come out.

Clearly this is not the case and this is where it would be useful to hold a discussion about what mathematical proof means and how we might begin to start constructing a proof for this example.

Solution

There are multiple proofs for this example. Here is one of the more straightforward proofs.

Let n be the first number
The next 4 will therefore be

n+1, n+2, n+3, n+4,
= 5n +10
= 5(n +2)

Which is always divisible by 5, thanks to the factor of 5.

Grandpa & Tom

Teacher Notes

Strand: Algebra
Group: Equations
Suggested Age: 11+

Pupils should be able to form an equation from a word problem and solve an equation with an unknown on both sides.
This can be solved without algebra by using a trial and improvement method.
Discussion point to look at the different approaches.

Solution

Let Tom's age now be x, so Grandpa's age must be $4x$.
If we consider 5 years ago then:

$4x - 5 = 5(x - 5)$

$4x - 5 = 5x - 25$

$20 = x$ (Tom)

And Grandpa $= 4x = 80$

Answer is therefore 80 + 20 = **100**

Index Indecision

Teacher Notes

Strand: Number
Group: Numbers and the Number System
Suggested Age: 11 - 16

This problem can suit different levels of difficulty. It can be used as a vehicle for practising the evaluation of powers and indices with students writing down what they notice.

Encourage students to hypothesise e.g. the result is always larger when the large number is used as the index number (the green box).

Encourage students to consider exceptions. What happens when zeros and ones are used?

This can be extended into using negative numbers. When negative numbers are used as the index number, the difference between odd and even becomes more important than size.

Mersenne Primes

Teacher Notes

Strand: Number
Group: Numbers and the number system
Suggested Age: 11+

Mersenne refers to French monk Marin Mersenne (17th Century)

Solution

$p = 2 \longrightarrow m_p = 2^2 - 1 = 3 \quad \therefore\ m_2 = 3$

$p = 3 \longrightarrow m_3 = 2^3 - 1 = 7$

$p = 5 \longrightarrow m_5 = 2^5 - 1 = 31$

$p = 11 \longrightarrow m_{11} = 2^{11} - 1 = 2047$

With some work on possible factors, pupils will be able to show that

$m_{11} = 23 \times 89$

$m_{11} \neq$ prime

Teacher Notes

Strand: Number, Algebra
Group: Trial and Improvement, Algebraic Manipulation, Equations
Suggested Age: 11 to 16

This problem is best solved by forming an equation to represent the situation and then solving it. Alternatively pupils can use trial and improvement to find a solution.

Solution

58 correct answers.

Trial and improvement.

Guess the number of correct answers, work out the total score, then adapt your guess to make the total score closer to 190.

By algebra

Call the number of correct answers A.

Total score	=	(4 × A) + (-1 × (100 - A)	=190
	=	4A + A - 100	=190

Therefore	5C = 290
and	A = 58

One Minute Challenge 2

Teacher Notes

Strand: Number, Geometry And Measures, Algebra, Probability And Statistics
Group: Numbers and the number system, Probability, Statistics, Formulae, Properties of 2D shapes, Trigonometry
Suggested Age: 11+

This is a quick fire quiz on general Mathematical knowledge covering facts related to geometry, probability, number and data. This quiz aims to get students to remember facts and concepts when they see them out of context in a random situation. Teachers should time this activity so that it lasts for no longer than one minute, giving six seconds per fact. The ability to quickly recall mathematical facts is necessary in future time pressured situations such as exams and tests. The capitalised letters in each statement should be replaced with words. This will get students thinking, and for the final challenge to make up two further questions of their own, students should use the questions as stimuli in order to arrive at a related statement, examples are provided below.

Solution

- 8 is the **S R** of **SF** – 8 is the square root of sixty four
- 180 **D** in a **T** – 180 degrees in a triangle
- πd is the **C** of a **C** – πd is the circumference of a circle
- 360 **D** in a **P C** – 360 degrees in a pie chart
- $c^2 = a^2 + b^2$ is **P T** – $c^2 = a^2 + b^2$ is Pythagoras' Theorem
- 10 **S** in a **D** – 10 sides in a decagon
- A **S** is a **R P** – A square is a regular polygon
- The **P** of **H** or **T** is $\frac{1}{2}$ – The probability of heads or tails is $\frac{1}{2}$

Examples of other statements that students may arrive at:

- 9 is the **SR** of **EO** – 9 is the square root of eighty one
- πr^2 is the **A** of a **C** – πr^2 is the area of a circle
- 8 **S** in an **O** – 8 sides in an octagon

One Minute Challenge 3

Teacher Notes

Strand: Number, Geometry and Measures, Probability and Statistics, Algebra
Suggested Age: 11+

This is a quick fire quiz on general Mathematical knowledge covering facts related to geometry, measure, number and data. This quiz aims to get students to remember facts and concepts when they see them out of context in a random situation. Teachers should time this activity so that it lasts for no longer than one minute, giving six seconds per fact. The ability to quickly recall mathematical facts is necessary in future time pressured situations such as exams and tests. The emboldened letters in each statement should be replaced with words. This will get students thinking, and for the final challenge to make up two further questions of their own, students should use the questions as stimuli in order to arrive at a related statement, examples are provided below.

Solution

- 83 is a Prime Number.
- $\frac{2}{3}$, $\frac{14}{21}$ and $\frac{38}{57}$ are Equivalent Fractions.
- A cube (or cuboid) has 8 Vertices.
- 215° is a Reflex Angle.
- 6 is the Cube Root of 216.
- 1000 millilitres in a Litre.
- $\frac{1}{2} \times b \times h$ is the Area of a Triangle.
- 540 Degrees in a Pentagon.

Examples of other statements that students may arrive at:

- 64 is a S N - 64 is a Square Number
- 180 D in a T – 180 Degrees in a Triangle
- 40° is an A C – 40 degrees is an Acute Angle

One Minute Challenge 4

Teacher Notes

Strand: Number, Geometry and Measures, Probability and Statistics, Algebra
Suggested Age: 11+

This is a quick fire quiz on general Mathematical knowledge covering facts related to geometry, measure, number and data. This quiz aims to get students to remember facts and concepts when they see them out of context in a random situation. Teachers should time this activity so that it lasts for no longer than one minute, giving six seconds per fact. The ability to quickly recall mathematical facts is necessary in future time pressured situations such as exams and tests. The emboldened letters in each statement should be replaced with words. This will get students thinking, and for the final challenge to make up two further questions of their own, students should use the questions as stimuli in order to arrive at a related statement, examples are provided below.

Solution

- $A = \frac{1}{2}(a + b)h$ is the Area of a Trapezium
- 1000kg = 1 Tonne
- 36 is a Square Number and an Even Number
- This is a Cyclic Quadrilateral.
- Parallel Lines never meet.
- 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 are Factors of 48 and 2 is also a Prime Factor
- 1, 3, 6 and 10 are the first four Triangle Numbers
- An Isosceles Triangle has two Equal Angles and two Equal Sides

Examples of other statements that students may arrive at:

- An ET has three E A and three E S – An equilateral triangle has three Equal Angles and three Equal Sides
- $\frac{1}{2} \times b \times h$ is the A of a T – $\frac{1}{2} \times b \times h$ is the Area of a Triangle

One Minute Challenge 5

Teacher Notes

Strand: Number, Geometry and Measures, Probability and Statistics, Algebra
Suggested Age: 11+

This is a quick fire quiz on general Mathematical knowledge covering facts related to geometry, measure, number and data. This quiz aims to get students to remember facts and concepts when they see them out of context in a random situation. Teachers should time this activity so that it lasts for no longer than one minute, giving six seconds per fact. The ability to quickly recall mathematical facts is necessary in future time pressured situations such as exams and tests. The emboldened letters in each statement should be replaced with words. This will get students thinking, and for the final challenge to make up two further questions of their own, students should use the questions as stimuli in order to arrive at a related statement, examples are provided below.

Solution

- 124º is an OBTUSE ANGLE
- Shapes A and B are CONGRUENT
- DIAMETER is the distance across the centre of a CIRCLE
- 168 HOURS in a WEEK
- 180º is the ANGLE of a STRAIGHT LINE
- This is a COMPOUND (COMPOSITE) SHAPE
- A VECTOR has MAGNITUDE and DIRECTION
- 19 × 7 × 12 × 0 × 5 × 23 = ZERO

Examples of other statements that students may arrive at:

- 46º is an A A - 46 o is an acute angle
- R = $\frac{1}{2}$ D – Radius is half diameter
- An O has 8 S – An octagon has 8 sides

Square Boxes

Teacher Notes

Strand: Number
Group: Numbers and the Number System & Number Theory
Suggested Age: 11 and up

This problem consolidates square numbers but is much more aimed at building logical thought and strategy.

Solution

9, 36, 25, 196, 1, 4

Ted & Tod

Teacher Notes

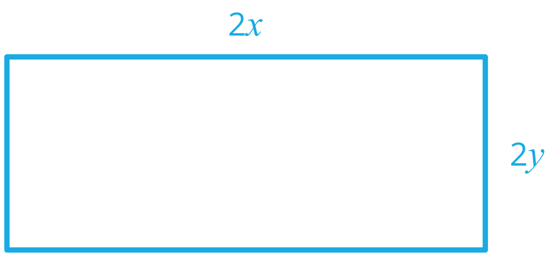
Strand: Algebra
Group: Equations
Suggested Age: 11+

Pupils should be able to form an equation from a word problem and solve an equation with an unknown on both sides.
This can be solved as a simultaneous equation problem, although students may not see that straight away as it is a word problem. Many students may prefer to use a 'solution by trial' method so this could make a good basis for discussion – 'trial' versus algebraic method.
Some students may recognise it is a simultaneous equation and may need help with forming the equations in the first instance, they should be encouraged to make a sketch. A discussion as to which method to use and around the proportional reasoning at the end of solution 2) would be beneficial.

Solution

Answer is 60cm

Let the original rectangle have sides $2x$ cm and $2y$ cm, where $2x > 2y$



Perimeter = $4x + 4y$
We can say Ted has rectangles x by $2y$, which have perimeters of $\mathbf{2x + 4y = 40}$
Tod has rectangles $2x$ by y, which have perimeters of $\mathbf{4x + 2y = 50}$

1) We can say that $2x = 40 - 4y$ from Ted's rectangles
Substitute into Tod's rectangle so, $2(40 - 4y) + 2y = 50$
$80 - 8y + 2y = 50$
$30 = 6y$
$y = 5$
Therefore: $2x + 4(5) = 40, \quad x = 10$
Original perimeter of $4x + 4y = 4(10) + 4(5) =$ **60cm**

2) Or we can add together both Ted and Tod's rectangles to give us $6x + 6y = 90$
Therefore, $4x + 4y = \frac{2}{3}(6x + 6y)$
Perimeter must be $\frac{2}{3}$ of 90 = **60cm**

Teacher Notes

Strand: Number
Group: Proportional Reasoning
Suggested Age: 12+

What does $\frac{4}{5}$ mean? It's a Fraction
Can they draw it?
It means 4 out of 5?

What does 4:5 mean? It's a Ratio.
For every 4 of one there are 5 of another – ie 4 red counters to 5 blue counters
So how many counters are there altogether … 9

Solution

$\frac{4}{9}$ means we are looking at 4 out of 5 equal parts.
4:5 means a total of 9 shares have been divided so that the first share is 4 of the 9 shares $\frac{4}{9}$ and the second is 5 of the 9 $\frac{5}{9}$

How Much For Smile?

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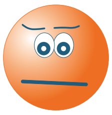
Teacher Notes

Strand: Algebra
Group: Equations
Suggested Age: 12+

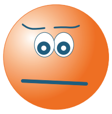
This is a simultaneous equation problem, although students may not see that straight away as it is abstract. Many students may prefer to use a 'solution by trial' method so this could make a good basis for discussion – 'trial' versus algebraic method.

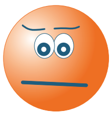
Some students may recognise it is a simultaneous equation and may need help with which two equations to use. A discussion as to why it is best to use an equation with two unknowns and not three would be beneficial here.

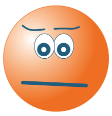
This could be extended to find the values of the other two smilers.

 = 4  = 5

Solution

Using the third column: 2  +  =13 (1)

Using the second row:  + 2  =11 (2)

(2) × by 2 = 2  + 4  =22 (3)

(3) - (1) = 3  =9

so,  =3

Penny's Pig Pen

Teacher Notes

Strand: Geometry And Measures
Group: Area, Perimeter and Volume
Suggested Age: 12+

If pupils have come across this sort of problem in the past they may know to use circular shaped pen. From this point it is a case of using the relationships between the circumference, radius and area of a circle – as seen in the solution.

Pupils may not, however, consider a circle, and instead experiment with rectangles, squares and triangles. It could also be a good activity to ask pupils what area of pen can be created with different shapes.

With a few prompts, pupils can begin to see the relationships between area and perimeter. Perhaps make them start with a rectangle. They have 40 metres to play with, and as opposite sides of a rectangle are equal, they effectively have 20 metres to play e.g. a 19m x 1m rectangular pen – what is the area? What about an 18m x 2m pen?

Pupils will see, through this method, that the closer they get to a square 10m x 10m pen, the larger the area gets. At this point you can highlight to pupils that regular shapes will always have the greatest area for a given perimeter, when compared to their irregular counterparts. From here there are two ways pupils could be led to the solution of a circular pen.

The first method may be through observation; highlight to them how mathematics is not just something in books, but all around us. Take inspiration from nature, which likes to be as efficient as possible - water droplets and soap bubbles for example, will always contain the most amount of rain or air, with the smallest possible surface area. Applying this logic, pupils may realise that a circle is the most efficient perimeter-to-area shape in two dimensions.

The second method is through logical progression. You could show the pupils what the area of the pen would be if Penny made the pen a regular triangle, then a regular square, then a regular hexagon etc. Each time we add a side to the shape of the pen, the area increases. So what would happen if we kept adding sides? We would have a shape tending towards a circle until it was in fact a circle, because each piece of fence, no matter where you measured from, would be the same distance from the center of the pen.

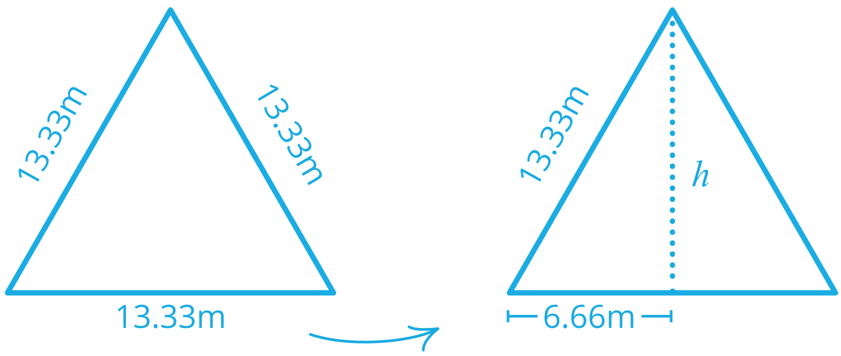
Extra: you may wish to discuss the practicalities of a circular pen – what if Penny wants to expand one day, or connect more pens for different animals. Perhaps look into how well circles tessellate for future expansion.

Penny's Pig Pen

Solution

Area of a triangular pen:

Total perimeter = 40m Therefore 40 ÷ 3 = 13.33m for each side of fence



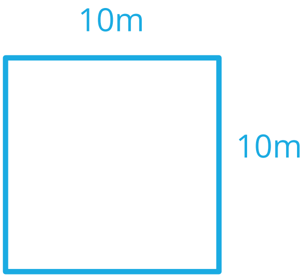
$6.66^2 + h^2 = 13.33^2$

$h^2 = 133.33$

$h = 11.55\text{m}$

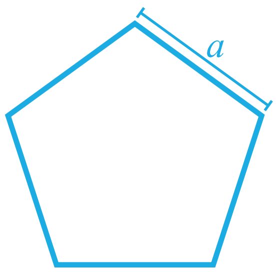
$A = \frac{h\,b}{2}$ → $A = \frac{11.55 \times 13.33}{2}$ → $A = 76.96\text{m (2dp)}$

Area of square pen:



$A = 10 \times 10 = 100\text{m}$

Area of a pentagonal pen:



$A = \frac{1}{4}\sqrt{5(5+2\sqrt{5})}\; a^2$

$A = 110.11\text{m (2dp)}$

Area of a circular pen:

Circumference = 40m

Using $C = 2\pi r$ we can work out

$r = \frac{40}{2\pi} = 6.37$

Using $A = \pi r^2$

$A = 127.32\text{m (2dp)}$

Special Whopper Sundae

Teacher Notes

Strand: Number
Group: Number Theory
Suggested Age: 12+

This is a lowest common multiple problem.
Some pupils may just complete multiplication tables for all the numbers until they find the first common one. If they do it this way it would be a point for discussion as to whether they need to do both 2 and 4.

Solution

The LCM Lowest Common Multiple of 2, 3, 4, 5 and 6 is required.
2, 3 and 5 are prime
4 = 2 x 2
6 = 2 x 3
Therefore, LCM = 2 x 2 x 3 x 5 = 60 days

Isosceles Triangle

From ③

$x = 2y$

$x = 2(36)$

$x = 72$

Substitute into ② to check

② $3x - y = 180$

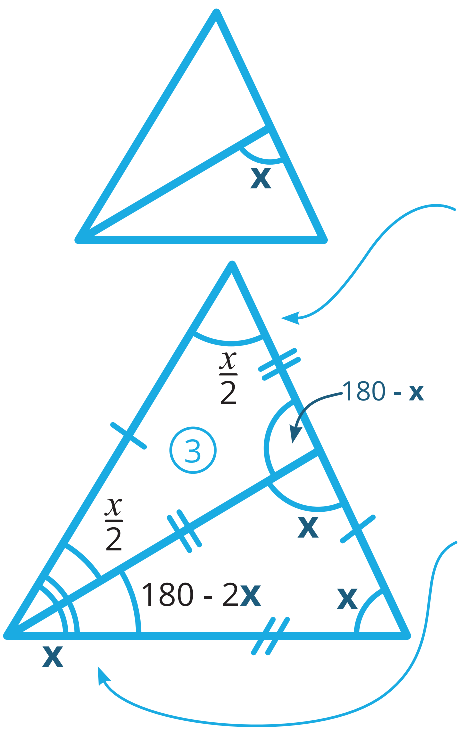
$3(72) - 36 = 180$

$210 - 36 = 180$

$180 = 180$ ✓

6. With **x** and **y** known, substitute into original expressions to find the size of all angles.

Solution 2



1 – As before

2 – Define ALL other angles in teams of **x**

This angle is found by looking at the base angles of triangle ③ call it angle **a**

$a + a + 180 - x = 180$

$2a - x = 0$

$2a = x$

$a = \frac{x}{2}$

3 – Use these three angles to build one equation in one unknown

$x + \frac{x}{2} + 180 - 2x$

4 – Solve

$x = \frac{x}{2} + 180 - 2x$

$3x = \frac{x}{2} + 180$

$6x = x + 180$

$5x = 180$

$x = 72$

5 – All other angles can now be calculated by substitution

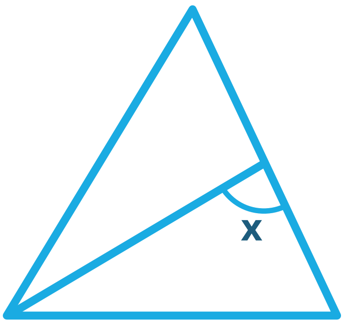
Isosceles Triangle

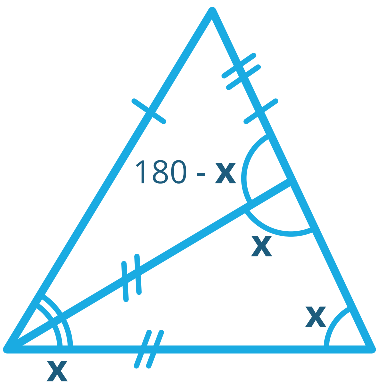
Teacher Notes

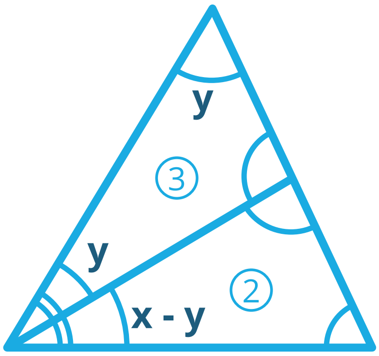
Strand: Algebra, Geometry
Group: Algebraic notation, Algebraic manipulation, Equations, Modelling, Geometric notation, Angles
Suggested Age: 13 - 18

This problem provides an opportunity for algebraic modeling of a complex problem.
There are several ways of solving it. Two are provided below:
Method 1 is more complex to solve, requiring 2 – 3 equations, and higher level simultaneous equations.
But, once the obvious angles have been identified, it can be tricky to spot that $y = \frac{x}{2}$ making the model from method 2 more difficult for students to spot

Solution







1. Start by labeling an angle as unknown

"An unknown quantity can be labeled with a letter"

2. Define other angles in terms of **x**

3. Define remaining angles in terms of **y**

4. Use three triangles to build 2-3 equations in **y** and **x**

① $y + x + x = 180$
$y + 2x = 180$

② $x + x + (x - y) = 180$
$3x - y = 180$

③ $y + y + (180 - x) = 180$
$2y - x + 180 = 180$
$2y - x = 0$
$2y = x$

5. Solve two of the equations simultaneously

③ → ①

$x = 2y$ → $y + 2x = 180$
$y + 2(2y) = 180$
$5y = 180$
$y = 36$

Oak Tree Park

Teacher Notes

Strand: Geometry
Group: Trigonmetry
Suggested Age: 13 - 16

The question can be made easier or harder by changing the destinations.

Students may attempt the question by using a scale drawing, Pythagoras' Theorem or trigonometry.

Instead of being given the map, students could be given the bearings and distance and asked to draw the map.

Students could be asked the scale of the map.

Solution

a) The large oak is 156.1m from the statue. Using Pythagoras' Theorem, the distance between the oak and the fountain is 231.02m.

She cycles from the entrance to the statue, then to the large oak, then to the fountain and finally out of the park via the statue again. She has travelled 812.82m in the park.

b) She has travelled 4.6km.

Peg of Best Fit

Teacher Notes

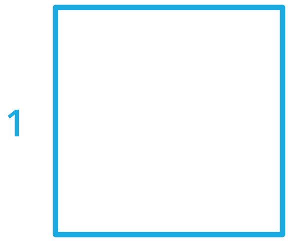
Strand: Geometry and Measures
Group: Area, Perimeter and Volume
Suggested Age: 13 and up

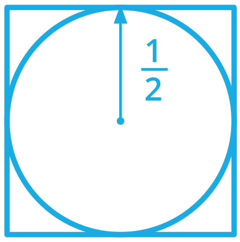
This problem requires an understanding of the area of a circle, Pythagoras' Theorem and proportion and percentages.

Solution

The round peg in the square hole fills a greater percentage of the space.

Round Peg - Square Hole



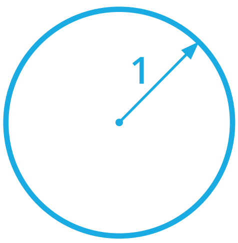


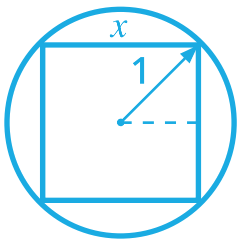
Area: 1 $\quad \frac{\pi}{4}$

Percentage of circle area to square area

$$\frac{\pi/4}{1} \approx 78.5\%$$

Square Peg - Round Hole





Area: π

$$1^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2$$

$$1 = \frac{2x^2}{4}$$

$$x = \sqrt{2}$$

$$\text{Area} = x \times x = 2$$

Percentage of square area to circle area

$$\frac{2}{\pi} \approx 63.7\%$$

Vinegar Volume

Teacher Notes

Strand: Geometry And Measures
Group: Area, perimeter and volume
Suggested Age: 13+

To solve this problem, students need to know how to find the area of a circle and the volume of cylinder. Students need to find a logical approach to solving problem related to the volume of a bottle of vinegar that has a tapered neck. All of the facts that students need are provided, but can students figure out how to use those facts?

Solution

You are looking for the internal volume, so firstly, subtract the thickness of the glass to find the internal radius of the base.

The diameter is 0.5cm less each side.

Total wall thickness = 0.5 x 2 = 1cm
Internal diameter = 6 – 1 = 5cm
Radius = Diameter ÷ 2
= 5 ÷ 2
= 2.5

The internal radius is 2.5cm

Now work out the cross sectional area:

$$A = \pi r^2$$
$$A = \pi \times 2.5^2$$
$$A = 19.63495408$$

You can find the volume of the bottle that is taken up by the liquid because the bottom of the bottle is cylindrical. Don't forget to take off 0.5cm to allow for the thickness of the base.

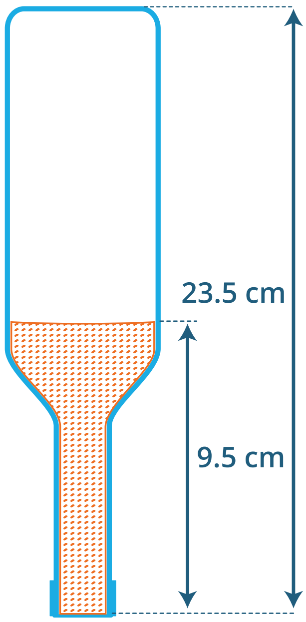
8.5 – 0.5 = 8cm

Vinegar Volume

Volume of a cylinder = Area of cross section x height

V = 8 x 19.63495408

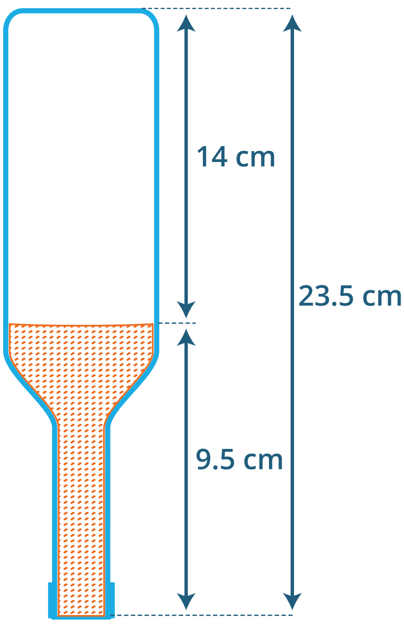
V= 157.0796327



Now here is the trick, how to find the volume remaining in the bottle and its neck. The key is the depth of liquid when the bottle is upturned

If you turn the bottle upside down, you can measure the space left above the liquid when upturned, or in this case you can subtract from the height.

You don't need to take anything off to allow for the glass at the top because the top of the bottle is empty (or how else would you pour from it?) However, you do need to take off 05.cm to allow for the base.



14 – 0.5 = 13.5cm

13.5cm is the depth of the upturned bottle occupied by air, you can work out the volume of this space.

Volume of a cylinder = Area of cross section x height

V = 13.5 x 19.63495408

V = 265.0718801

Adding together the two volumes that you have found will give you the volume of the whole bottle!

V = 157.0796327 + 265.0718801

V = 422.1515128cm^3

1cm^3 = 1ml

Volume of the bottle = 422ml

'Broad'-band

Teacher Notes

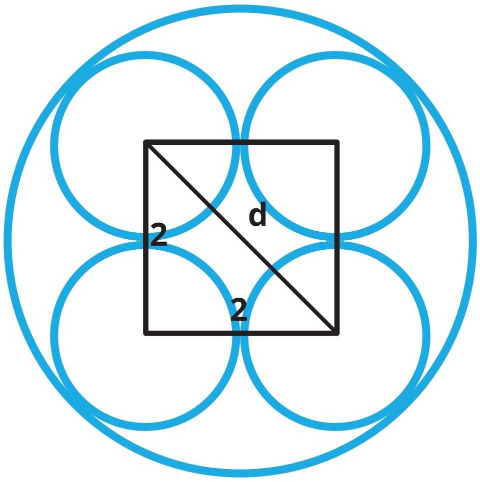
Strand: Geometry and Measures
Group: Geometry, Trigonometry (Circles and Pythagoras)
Suggested Age: 14 - 16

This problem requires knowledge of circles and Pythagoras to solve it. The first solution pupils should find very quickly by looking at the picture. The second solution is much more difficult because of the spacing change. Pupils need to realise they need to know the distance between two centres of the smaller circles to find the total distance across.

Solution

The duct for two cables must have a diameter of 4 cm as it is two 2cm diameters side by side.

Four Cables



Using the Pythagorean Theorem, $d^2 = 2^2 + 2^2$, giving $d = \sqrt{8} \approx 2.83$ cm. Therefore, the duct diameter will be $\sqrt{8} + 2 \approx 4.83$ cm.

How Many Ways?

Teacher Notes

Strand: Number
Group: KS4
Suggested Age: 14 - 16

The aim of the problem is to engage students in a Permutation problem before they have met this at AS Level.
Initially pupils will often start with the idea that there are 10 ways, but they quickly abandon this and start exploring ideas such as 10 x 10 = 100 ways.

Solution

Mathematically the solution is 10! = 3,628,800 different permutations.

The reason is, when we choose the first book for the shelf, we have 10 options.
Once the first book is on the shelf, there are 9 ways of choosing the second book, so far there are 10 x 9 = 90 ways of choosing the first two books.
Once the first & second books are on the shelf there are 8 ways of choosing the 3rd.
There are 10 x 9 x 8 = 720 ways of picking the first 3 books.

This continues until the last book, meaning there are 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 3628800 ways of ordering these books.

10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 is also called 10 factorial which can be written as 10!

One Minute Challenge 6

Teacher Notes

Strand: **Number**, **Geometry and Measures**, **Probability and Statistics**, **Algebra**
Suggested Age: **14+**

This is a quick fire quiz on general Mathematical knowledge covering facts related to geometry, measure, number and data. This quiz aims to get students to remember facts and concepts when they see them out of context in a random situation. Teachers should time this activity so that it lasts for no longer than one minute, giving six seconds per fact. The ability to quickly recall mathematical facts is necessary in future time pressured situations such as exams and tests. The emboldened letters in each statement should be replaced with words. This will get students thinking, and for the final challenge to make up two further questions of their own, students should use the questions as stimuli in order to arrive at a related statement, examples are provided below.

Solution

- A line just touching the edge of a CIRCLE is a TANGENT
- 7 is the SQUARE ROOT of FORTY NINE
- y = mx + c is the EQUATION of a STRAIGHT LINE
- 169 is a SQUARE NUMBER and an ODD NUMBER
- A TETRAHEDRON is a PLATONIC SOLID
- Side c is the HYPOTENUSE
- This is the INTERSECTION of a VENN DIAGRAM
- 77 is not a PRIME NUMBER

Examples of other statements that students may arrive at:

- A C is a PS – A cube is a platonic solid
- 12 is the SR of a H an F F – 12 is the square root of a hundred and forty four.

Rhombus Overlap

Teacher Notes

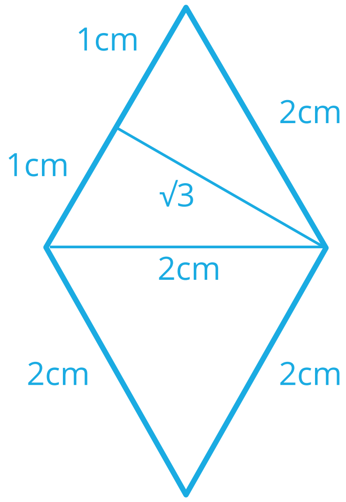
Strand: Geometry And Measures
Group: Measures
Suggested Age: 14+

Pupils will need to recognise that the rhombus is made up of two equilateral triangles and then use Pythagoras' Theorem to calculate the height of each triangle.

Most students will find the area of one triangle and then double the answer.
It is quicker, and more elegant, to use the formula for the area of a rhombus (base x height) if you realise that the height of the triangle is also the height of the rhombus.

Solution

Answer: $2\sqrt{3}$ cm^2



The rhombus is made up of 2 equilateral triangles, each of side length = 2 cm.
Using Pythagoras' Theorem, the height of the rhombus = $\sqrt{3}$ cm
Pupils may recognise the standard 1, 2, $\sqrt{3}$ (i.e. 30° / 60° / 90°) triangle.
Therefore the area of the rhombus = base x height = $2\sqrt{3}$ cm^2

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Square Within a Square

Teacher Notes

Strand: Geometry and Measures
Group: Area, Perimeter and Volume
Suggested Age: 14 and up

Students might need reminding about how to use algebraic notation and area. Students should be encouraged to explore the problem without too much direction initially. However, if they become stuck, probing questions can be used to assist with their progress, such as:

- What would the area of one of the triangles be?
- Do we need to consider the triangles at all?

Ideally the area will be expressed in two different ways which will then allow a discussion about proving Pythagoras' Theorem.

Solution

a) $a^2 + b^2 + 2ab$

b) which then can be written as:

$$2ab + c^2 = a^2 + b^2 + 2b$$

$$c^2 = a^2 + b^2$$

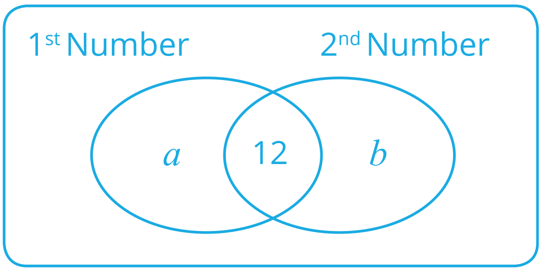
Think of Two Numbers

Teacher Notes

Strand: Number
Group: Number Theory, Fractions and Decimals, Proportional Reasoning
Suggested Age: 14+

Pupils will need to be fluent at breaking numbers into their prime factors and using these to find the HCF and the LCM.
Using a Venn diagram helps to visualise how the HCF and LCM are connected and therefore helps to show that we need to split $\frac{4620}{12} = 385$ into prime numbers (385 = 5 x 7 x 11)
As there are more than one way of placing 5, 7 and 11 on the Venn Diagram, we can see there are different possible values for 'a' and 'b'.
Each possible answer can be tried until we locate the positions that give the correct sum for the two numbers (students will quickly realise the correct positions of 5, 7 and 11 once they have tried an initial possible solution).

Solution



Highest Common Factor = 12
Lowest Common Multiple = 4620
Therefore, $a \times 12 \times b = 4620$

So $\frac{4620}{12} = 385 = a \times b$

Breaking 385 into its prime factors gives:
385 = 5 x 7 x 11
So we have a few combinations to try…
1st number = 5 x 12 = 60, 2nd number = 12 x 77 = 924 – discard as the sum is too big
1st number = 7 x 12 = 84, 2nd number = 12 x 55= 660 – discard as the sum is still too big
1st number = 5 x 7 x 12 = **420**, 2nd number = 12 x 11 = **132**
420 + 132 = 552 – **Correct Answer**

Amy is thinking of 420 and 132.